- Theorem: Suppose that $F(x)$ is an antiderivative of $f(x)$ and $G(x)$ is an antiderivative of $g(x)$. Then, for any constants $a$ and $b$, one antiderivative of $a f(x)+b g(x)$ is $a F(x)+b G(x)$.

In order to see that $a F(x)+b G(x)$ is an antiderivative of $a f(x)+b g(x)$, we need to check that

$$
\frac{d}{d x}(a F(x)+b G(x))=a f(x)+b g(x)
$$

Since $F^{\prime}(x)=f(x)$ and $G^{\prime}(x)=g(x)$, then by combining Theorem 2.3.1 (i) and (iii), we see that:

$$
\frac{d}{d x}(a F(x)+b G(x))=a \frac{d}{d x}(F(x))+b \frac{d}{d x}(G(x))=a f(x)+b g(x) .
$$

- Theorem: For $x \neq 0, \frac{d}{d x}(\ln |x|)=\frac{1}{x}$.

Recall that

$$
|x|= \begin{cases}x & \text { if } x>0 \\ -x & \text { if } x<0\end{cases}
$$

If $x>0$, then $\ln |x|=\ln (x)$, and so

$$
\frac{d}{d x}(\ln |x|)=\frac{d}{d x}(\ln (x))=\frac{1}{x}
$$

On the other hand, if $x<0$, then $\ln |x|=\ln (-x)$, and so

$$
\frac{d}{d x}(\ln |x|)=\frac{d}{d x}(\ln (-x))=(-1) \frac{1}{-x}=\frac{1}{x} .
$$

Thus in either case, $\frac{d}{d x}(\ln |x|)=\frac{1}{x}$.

## - Table of antiderivatives:

In each case, $F(x)$ represents an antiderivative of $f(x)$ :

| $f(x)$ | $F(x)$ | $f(x)$ | $F(x)$ |
| :--- | :--- | :--- | :--- |
| $x^{r}, r \neq 1$ | $\frac{x^{r+1}}{r+1}$ | $\sec (x) \tan (x)$ | $\sec (x)$ |
| $\sin (x)$ | $\cos (x)$ | $\csc (x) \cot (x)$ | $-\csc (x)$ |
| $\cos (x)$ | $-\sin (x)$ | $e^{x}$ | $e^{x}$ |
| $\sec ^{2}(x)$ | $\tan (x)$ | $x^{-1}=\frac{1}{x}, x \neq 0$ | $\ln \|x\|$ |
| $\csc ^{2}(x)$ | $\cot (x)$ |  |  |

- Theorem: Suppose that $F$ and $G$ are both antiderivatives of a function $f$ on an interval $I$. Then

$$
G(x)=F(x)+c,
$$

for some constant $c$.

This is really just a restatement of Corollary 2.9.1, which states that if $g^{\prime}(x)=f^{\prime}(x)$ for all $x$ in some open interval $I$, then for some constant $c, g(x)=f(x)+c$ for all $x \in I$.
To see this, we have only to recall that since $F$ and $G$ are both antiderivatives of $f$ on the interval $I$, it follows that on the interval $I, G^{\prime}(x)=f(x)=F^{\prime}(x)$.

Thus by Corollary 2.9.1, there is some constant $c$ such that $G(x)=F(x)+c$.

